Biased Technological Change and Employment Reallocation*

Zsófia L. Bárány† Christian Siegel‡

8 October 2019

Abstract

To study the drivers of the employment reallocation across sectors and occupations between 1960 and 2017 in the US we present a model where technology evolves at the sector-occupation cell level. Drawing on key equations of the production side we infer technologies directly from the data. We assess the magnitude of sector- and occupation-specific components in technological change and study their consequences for labor market outcomes in general equilibrium where occupational choice and demands for sectoral outputs change endogenously with technology. Our findings indicate a major role for occupation-specific technological changes.

Keywords: biased technological change, structural change, employment polarization

JEL codes: O41, O33, J24

*Parts previously circulated as ‘Disentangling Occupation- and Sector-specific Technological Change’. We wish to thank Francesco Caselli, Georg Duerncker, Tim Lee, Miguel León-Ledesma, Guy Michaels, Mathan Satchi, Ákos Valentinyi as well as numerous seminar and conference participants.
†Sciences Po, CEPR and ROA. Email: zsobia.barany@sciencespo.fr
‡University of Kent, School of Economics and Macroeconomics, Growth and History Centre and ROA. Email: c.siegel@kent.ac.uk
1 Introduction

There have been substantial changes in the structure of employment over recent decades in most developed countries. Most economies are undergoing structural change, whereby labor reallocates across sectors, while at the occupational level labor markets have been polarizing, with employment shifting out of middle-earning routine jobs to low-earning manual and high-earning abstract jobs. There is a wide consensus in the literature that the main driver of each of these patterns separately is biased technological change. Technological change biased across sectors is the main explanation for structural change, technological change biased across factors of production (occupations/tasks) is a prominent explanation for job polarization.\footnote{See for example Kongsamut, Rebele, and Xie (2001), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008) for structural change, Autor, Katz, and Kearney (2006), Goos and Manning (2007), Autor and Dorn (2013), Goos, Manning, and Salomons (2014), Michaels, Natraj, and Van Reenen (2014) for polarization.} The benefits of such non-neutral technological progress are not equally distributed, with some workers displaced or their skills becoming less valuable in the labor market. These distributional impacts have generated an interest in policies to counteract the adverse effects of technological change. To devise and implement such policies, the biases of technological change and their implications for the labor market needs to be understood.

Figure 1 shows that the evolution of sectoral and of occupational employment shares are closely connected. Virtually all of the decline in the goods-producing sector’s employment share is due to a decline in routine employment in this sector. Conversely, most of the increase in high-skilled service employment is a rise in employment in abstract occupations in that sector. Because of this tight connection between employment reallocations across sectors and across occupations, models that do well in matching sectoral outcomes do also quite well in replicating certain aspects of occupational outcomes, and vice versa, models that imply polarization of occupational employment can also be consistent with the observed sectoral shifts of employment. However, this also poses a challenge for establishing what the true drivers of these phenomena are; is it technological change biased across industrial sectors or across tasks/occupations?

In this paper we shed light on this question using a general equilibrium model in
which we allow for technologies to evolve at the sector-and-occupation level, thereby not a priori imposing restrictions on how technological change is biased. Similarly to our previous work (Bárány and Siegel (2019)), we draw on key equations of the production side of this model together with data from the US Census and from the U.S. Bureau of Economic Analysis between 1960 and 2017 to extract sector-and-occupation specific productivity growth, and decompose these into occupation- and sector-specific components using a factor model. Crucially, in order to study the contribution of these components to labor market outcomes, we specify a general equilibrium model, where individuals’ occupational choice and the demand for sectoral output responds endogenously to changing technologies. We use this general equilibrium model to quantify the role of the various components via counterfactual simulations in the reallocation of employment across sectors and occupations, as well as in the evolution of occupational wages and of sectoral prices.

One strength of our fully flexible approach is that it does not impose any particular type of technological progress a priori. This departure from the recent literature is especially important due to the trends shown in Figure 1 which have not received much attention. In Bárány and Siegel (2018) we show that differences in productivity
growth across sectors, lead to polarization of wages and employment at the sectoral level, which in turn imply polarization in occupational outcomes because of differences in occupation intensities across sectors. Goos et al. (2014) argue conversely that differences in productivity growth across occupations together with differences in occupation intensity across sectors can lead to employment reallocation across sectors. In a similar vein, Duurnecker and Herrendorf (2016), Lee and Shin (2017) and Aum, Lee, and Shin (2018) show that differences in occupational productivity growth can generate both structural transformation and changes in occupational employment consistent with the data. Such restricted models load all differences in technological change on one type of factor, therefore not allowing to identify whether these differences arise indeed at the level of sectors or of occupations, and therefore are less suitable for informing policy.

We assume a CES production function in manual, routine, and abstract labor in each sector. Using firm optimality conditions we extract sector-and-occupation specific productivities from the data for a given value of the elasticity of substitution. Our approach is similar to Katz and Murphy (1992) and Krusell, Ohanian, Rios-Rull, and Violante (2000), which both specify CES production functions in skilled and unskilled labor. These papers estimate the elasticity of substitution between the factors of production under the assumption of a specific form of factor-augmenting technological change. Conversely, we fix the value of the elasticity of substitution, but allow sector-occupation technologies to evolve over time without restrictions.

We take these sector-occupation cell productivity changes and use a factor model to identify sector-specific and occupation-specific components. This is more rigorous than calibrating sector- and occupation-specific growth rates, as those would depend on the normalizations implemented. We find that sector- and occupation-components

\[ \text{Note that it is not possible to infer biased technological change from the data without a model.} \]

\[ \text{Factor models have been used for instance in Stockman (1988), Ghosh and Wolf (1997) and Koren and Tenreyro (2007). While all these papers run a factor model at the country-sector level, they use their estimates to decompose the volatility of a series at a higher level of aggregation. We, however, not only study a very different question, but build counterfactual cell productivity series based on our factor model estimates.} \]

\[ \text{Each separately cannot be calibrated, some sector-specific growth rates and/or some occupation-specific growth rates need to be normalized. It can be shown that different normalizations have different implications in terms of the role of sector- and occupation-specific productivity growth.} \]
jointly explain around 90 percent of the variation in cell productivity growth. From these components we construct counterfactual cell productivity series and assess their importance first in the observed cell productivity growth. We find that occupation-specific productivity growth by far is the most important. This establishes that most of productivity changes are biased across occupations and to a lesser extent across sectors, and that a non-trivial part of technology is specific to the sector-occupation cell.

Most importantly, we then evaluate the role of sector- and occupation-specific components in labor market outcomes. To do this we specify a general equilibrium model allowing us to conduct counterfactual exercises. In particular, we assume that a representative household chooses sectoral consumption in order to maximize a non-homothetic CES utility and that individuals optimally choose their occupation subject to idiosyncratic entry costs. We feed the counterfactual productivity paths into the model to determine how important each component is in explaining various outcomes of interest. We find that while qualitatively both the sector and the occupation productivity components generate employment and wage paths in line with the data, quantitatively the occupation component gets much closer. To explain the evolution of sectoral prices, both sector and occupation components are needed. For occupational income shares within sectors and employment shares at the cell level, the sector-only component has almost no effect, whereas the productivity component specific to the sector-occupation cell has a significant role. An implication of these findings is that policies targeting workers' occupational choice might be better at improving labor market outcomes than industrial policies.

The paper proceeds as follows: section 2 introduces the model and section 3 presents the data and the model parameterization. In section 4 we first identify sector- and occupation-specific components of productivity growth, and then analyze the role that each of these components play in our general equilibrium model. The final section concludes.
2 Model

We assume that there is a continuum of measure one of heterogeneous workers in the economy. Workers optimally select their occupation and can freely choose which sector of the economy to supply their labor in. This implies that in equilibrium there is a single wage rate in each occupation which is common across sectors. We further assume that the different types of labor are imperfect substitutes in the production process in each sector, and that each sector values these types of workers differently in production.

The three types of workers are organized into a stand-in household, which derives utility from consuming all types of goods and services, and maximizes its utility subject to its budget constraint. The economy is in a decentralized equilibrium at all times: firms operate under perfect competition, prices and wages are such that all markets clear.

We use this parsimonious static model to pin down how the sector-occupation-specific technologies change over time, which we then decompose into common factors, as described in section 4.1. This is similar to the approach we take in Bárány and Siegel (2019). Note, here we do not model non-labor inputs but the technology parameters subsume effects stemming from changes in inputs other than labor. Not modeling capital as a distinct input to labor-augmenting technologies is consistent with the view that technological change is embodied into capital. While the focus of our work in Bárány and Siegel (2019) is to account for sources of sectoral productivity growth, here we study the implications of various forms of technological change for labor market outcomes. To do this we specify a general equilibrium model allowing us to conduct counterfactual exercises where occupational choice and demands for goods and services respond endogenously to changing technologies.

2.1 Sectors and production

There are three sectors in the economy which respectively produce low-skilled services ($L$), goods ($G$), and high-skilled services ($H$). All goods and services are produced in perfect competition. Each sector uses only labor as input in its production, but
each combines all three types of occupations (manual, routine and abstract), with the following CES production function:

$$Y_J = \left[ (\alpha_{m,J} l_{m,J})^{\eta^{-1}} + (\alpha_{r,J} l_{r,J})^{\eta^{-1}} + (\alpha_{a,J} l_{a,J})^{\eta^{-1}} \right]^{\frac{1}{\eta}} \quad \text{for } J \in \{L, G, H\},$$  \hspace{1cm} (1)

where \(l_{o,J}\) is occupation \(o\) labor used in sector \(J\), \(\alpha_{o,J} > 0\) is a sector-occupation specific labor augmenting technology term for occupation \(o \in \{m, r, a\}\) in sector \(J\), and \(\eta \in [0, \infty)\) is the elasticity of substitution between the different types of labor. In the initial year \(\alpha_{o,J}\) reflects the initial productivity as well as the intensity at which sector \(J\) uses occupation \(o\), whereas any subsequent change over time reflects sector-occupation specific technological change. We do not make any assumptions about whether technological change occurs at the occupation or the sector level, but instead allow for \(\alpha_{o,J}\) to evolve freely over time without imposing any restrictions. 

All firms take prices and wages as given, and profit maximization implies that the optimal relative labor demand within a sector has to satisfy:

$$\frac{l_{m,J}}{l_{r,J}} = \left( \frac{w_r}{w_m} \right)^{\eta} \left( \frac{\alpha_{m,J}}{\alpha_{r,J}} \right)^{\eta-1},$$  \hspace{1cm} (2)

$$\frac{l_{a,J}}{l_{r,J}} = \left( \frac{w_r}{w_a} \right)^{\eta} \left( \frac{\alpha_{a,J}}{\alpha_{r,J}} \right)^{\eta-1}. \hspace{1cm} (3)$$

These equations show that it is optimal to use less of one occupational labor input compared to routine labor if that occupation’s wage relative to routine is higher. In addition, the occupational labor inputs’ technologies within a sector matter. The larger the term \(\left( \frac{\alpha_{o,J}}{\alpha_{r,J}} \right)^{\eta-1}\) is in a sector, the more manual compared to routine labor the sector employs optimally. So for example routinization, i.e. the replacement of routine workers by certain technologies, would be captured by an increase in \(\left( \frac{\alpha_{m,J}}{\alpha_{r,J}} \right)^{\eta-1}\) and in \(\left( \frac{\alpha_{a,J}}{\alpha_{r,J}} \right)^{\eta-1}\) in all sectors \(J\). 

---

5 We assume the same elasticity of substitution in all sectors since we do not want to confound changes in productivity that are specific to sectors with potential differences in elasticities.

6 Given the close link between the sectoral and the occupational reallocation of employment, which we discussed in the introduction, had we set up the production function allowing only for sector-specific or only for occupation-specific terms we would potentially have attributed changes to this one factor which are actually due to the other factor. Our approach circumvents this problem as we do not impose any a priori restrictions on the evolution of technologies.
The firm first order conditions also pin down the price of sector \( J \) output in terms of wage rates:

\[
\begin{align*}
p_J &= \left[ \alpha_{mJ}^{\eta-1} \frac{1}{w_m^{\eta-1}} + \alpha_{rJ}^{\eta-1} \frac{1}{w_r^{\eta-1}} + \alpha_{aJ}^{\eta-1} \frac{1}{w_a^{\eta-1}} \right]^{\frac{1}{1-\eta}}. 
\end{align*}
\] (4)

Finally using (2), (3) and (4) to express sector \( J \) output, optimal sectoral labor use can be expressed as:

\[
\begin{align*}
l_{mJ}^d &= \left[ \frac{p_J \alpha_{mJ}}{w_m} \right]^\eta \frac{Y_J}{\alpha_{mJ}}, 
\end{align*}
\] (5)

\[
\begin{align*}
l_{rJ}^d &= \left[ \frac{p_J \alpha_{rJ}}{w_r} \right]^\eta \frac{Y_J}{\alpha_{rJ}}, 
\end{align*}
\] (6)

\[
\begin{align*}
l_{aJ}^d &= \left[ \frac{p_J \alpha_{aJ}}{w_a} \right]^\eta \frac{Y_J}{\alpha_{aJ}}.
\end{align*}
\] (7)

### 2.2 Households – occupational choice and demand for goods

The economy is populated by a unit measure of workers, who each have an idiosyncratic cost for entering each occupation, but can freely move between the three sectors, low-skilled services, goods, or high-skilled services, implying that in equilibrium, occupational wage rates must equalize across sectors. The cost that individuals pay for entering an occupation is redistributed in a lump-sum fashion. Since the consumption decisions are taken by the stand-in household, individuals choose the occupation that provides them with the highest income. Thus an individual \( i \) chooses occupation \( o \) if

\[
w_o - \chi_i^o \geq w_k - \chi_i^k \quad \text{for any} \quad k \neq o, \quad k, o \in \{m, r, a\},
\]

where \( w_o \) is the unit wage in occupation \( o \) and \( \chi_i^o \) is individual \( i \)'s cost of entering occupation \( o \). Since only the cost differences matter, we define \( \chi_i^1 \equiv \chi_i^r - \chi_i^m \) and \( \chi_i^2 \equiv \chi_i^a - \chi_i^m \). The optimal occupational choice is summarized in Figure 2.

Given the optimal occupational choice the fraction of labor supplied in the three

\[\text{The full derivations can be found in Appendix A.2}\]
occupations is given by:

\[ l_m^e = \int_{w_r-w_m}^{\infty} \int_{w_a-w_m}^{\infty} f(\chi_1, \chi_2) d\chi_1 d\chi_2, \tag{8} \]

\[ l_r^e = \int_{-\infty}^{w_r-w_m} \int_{w_a-w_r+\chi_1}^{\infty} f(\chi_1, \chi_2) d\chi_1 d\chi_2, \tag{9} \]

\[ l_a^e = \int_{0}^{\min\{w_a-w_r+\chi_1, w_a-w_m\}} \int_{-\infty}^{\infty} f(\chi_1, \chi_2) d\chi_1 d\chi_2, \tag{10} \]

where \( f(\chi_1, \chi_2) \) is the joint probability density function of the occupational cost differences.

The workers are organized into a stand-in household, which collects all income, and makes utility maximizing choices in terms of sectoral consumption. The stand-in household solves the following problem:

\[ \max_{c_L, c_G, c_H} \left( c_L (c_L + \tau_L)^{\frac{\varepsilon-1}{\varepsilon}} + a_G c_G^{\frac{\varepsilon-1}{\varepsilon}} + a_H (c_H + \tau_H)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon}} \]

s. t. \( p_L c_L + p_G c_G + p_H c_H \leq l_m w_m + l_r w_r + l_a w_a \)

where \( \tau_L \) and \( \tau_H \) allow for non-homotheticity in consumption demands, and \( \varepsilon < 1 \), implying that goods and services are complements in consumption. We further as-
sume that \(a_L + a_G + a_H = 1\). The price of low-skilled services is denoted by \(p_L\), that of goods by \(p_G\), and that of high-skilled services by \(p_H\). Assuming that the household is rich enough to consume all types of goods and services (i.e. an interior solution), optimality implies the following demand schedule:

\[
C_L = \left( \frac{a_L}{p_L} \right)^\varepsilon \left( \frac{f_m w_m + f_r w_r + f_a w_a + p_L \bar{c}_L + p_H \bar{c}_H}{a_L p_L^{1-\varepsilon} + a_G p_G^{1-\varepsilon} + a_H p_H^{1-\varepsilon}} \right) - \bar{c}_L, \tag{11}
\]

\[
C_G = \left( \frac{a_G}{p_G} \right)^\varepsilon \left( \frac{f_m w_m + f_r w_r + f_a w_a + p_L \bar{c}_L + p_H \bar{c}_H}{a_L p_L^{1-\varepsilon} + a_G p_G^{1-\varepsilon} + a_H p_H^{1-\varepsilon}} \right), \tag{12}
\]

\[
C_H = \left( \frac{a_H}{p_H} \right)^\varepsilon \left( \frac{f_m w_m + f_r w_r + f_a w_a + p_L \bar{c}_L + p_H \bar{c}_H}{a_L p_L^{1-\varepsilon} + a_G p_G^{1-\varepsilon} + a_H p_H^{1-\varepsilon}} \right) - \bar{c}_H. \tag{13}
\]

### 2.3 Equilibrium

There are six markets in this economy: three labor markets, that of manual, routine and abstract labor; and three goods markets, that of low-skilled services, goods, and high-skilled services. There are six corresponding prices, out of which we normalize one without loss of generality, \(w_r = 1\). The equilibrium is then defined as a set of prices, \(w_m, w_a, p_L, p_G, p_H\), for which all markets clear.

Goods market clearing requires that \(Y_L = C_L, Y_G = C_G,\) and \(Y_H = C_H\). Note that sectoral prices depend on the endogenous occupational wage rates, \(w_m\) and \(w_a\), through (4), and hence can be written as \(p_J = p_J(w_m, w_a)\). Using this in (11), (12), and (13), sectoral demands also only depend on occupational wage rates, \(C_J = C_J(w_m, w_a)\). Then (5), (6), and (7) show that optimal occupation \(o\) labor use in sector \(J\) can be expressed as a function of manual and abstract wage rates:

\[
l_{aJ}^o(w_m, w_a) = \left[ \frac{p_J(w_m, w_a)\alpha_oJ}{\omega_o} \right] \eta \frac{C_J(w_m, w_a)}{\alpha_oJ} \quad \text{for } o \in \{m, r, a\} \text{ and } J \in \{L, G, H\}.
\]

The equilibrium then boils down to finding wage rates \(w_m\) and \(w_a\) such that the labor
markets clear:

\[ l^d_mL(w_m, w_a) + l^d_mG(w_m, w_a) + l^d_mH(w_m, w_a) = l^*_m(w_m, w_a), \]
\[ l^d_rL(w_m, w_a) + l^d_rG(w_m, w_a) + l^d_rH(w_m, w_a) = l^*_r(w_m, w_a). \]

3 Calibration

To evaluate how sector-occupation-specific technology evolved over time and to study their implications for labor markets, we parameterize the model. We need to calibrate the sectoral production functions, the distribution of the costs of entering the different occupations, and the utility function. In our model setup, there is a dichotomy that allows to back out the sector-occupation cell productivities from the data using only the production side. We therefore proceed in the following steps, similarly to Buera, Kaboski, Rogerson, and Vizcaino (2018). First, we compute cell productivities taking as given the occupational wage rates and employment shares, as well as the sectoral income shares, in order to match in each period the income share of different occupations within each sector, the relative sectoral prices, and the overall growth rate of the economy (as in Bárány and Siegel (2019)). Second, we calibrate the distribution of costs such that it allows us to match occupational employment shares and wages in the initial and final period. Finally, we calibrate the utility function such that the model matches the sectoral income shares in the initial and final period.

3.1 Calibration targets

We use US Census and American Community Survey (ACS) data between 1960 and 2017 from IPUMS, provided by Ruggles, Alexander, Genadek, Goeken, Schroeder, and Sobek (2010), to calculate occupational wage rates and occupational labor income shares within sectors, as well as each sector’s share in labor income.\footnote{If the manual and the routine labor markets clear, then the market for abstract labor clears as well due to Walras’ law.} For these cal-

\footnote{In our model the share of each sector in labor income and in value added is the same as there are no other factors of production. As our main focus in this paper is on labor market outcomes, we calibrate the model to match the each sector’s share in labor income, rather than in value added.}
culations, we categorize workers into our three sectors based on their industry code (`ind1990`), and into our three occupations based on a harmonized and balanced panel of occupational codes as in [Autor and Dorn (2013)] and [Bárány and Siegel (2018)].

We calculate the labor income share of occupation $o$ in sector $J$ as the ratio of total labor income of workers in occupation $o$ and sector $J$ relative to the total labor income of all workers in sector $J$:

$$\theta_{oJ} \equiv \frac{\text{earnings of occupation } o \text{ workers in sector } J}{\text{earnings of sector } J \text{ workers}}.$$

To get measures for the occupational wage rates we employ the following Mincer wage regression to control for workers’ observable skills,

$$\log w_{iot} = \delta_{ot} + \beta' X_{it} + \varepsilon_{iot}, \quad (14)$$

where $\delta_{ot}$ are occupation-time effects and $X_{it}$ is a vector of worker characteristics. From this regression we back out for each year $t$ a wage for occupation $o$ that is not confounded by changes in composition of worker characteristics, $X_{it}$. In particular, we run this regression on the Census/ACS data where the vector of worker characteristics $X_{it}$ is comprised of a third-order polynomial in potential experience (defined as age minus years of schooling minus 6), interacted with a gender dummy, as well as a dummy for foreign-born and non-white race. From the estimates of this regression we construct for each year the manual wage rate as $w_{mt} = e^{\delta_{mt} - \delta_{rt}}$ and abstract wage rate as $w_{at} = e^{\delta_{at} - \delta_{rt}}$, maintaining the normalization of $w_r = 1$ in all years. We measure wages in this way – rather than as the average hourly wage of all workers within an occupation – to limit the potential influence of compositional changes, for example due to differential changes in the demographic composition or in educational attainment of workers across occupations. This is similar to [Buera et al. (2018)], and it implies that all differences within an occupational group in hourly wages are due to differences in the endowment of efficiency units of labor. [10]

---

[10] Given that we do not explicitly model heterogeneity in efficiency labor across individuals, the way we model selection implies that selection into occupations is orthogonal to efficiency. This assumption allows us to take wages from the data (using the regression (14)) which is needed for inferring the cell-technology terms, as outlined in Section 3.2.
We can express occupational labor supply shares as:

\[ l_o = \frac{\text{earnings of workers in occupation } o}{\sum_o \frac{\text{earnings of workers in occupation } \tilde{o}}{w_{\tilde{o}}}} \]

these are equivalent to occupational labor supplies in the model, as total labor supply is normalized to one. Figure 7 in the Appendix shows the evolution of these implied sector-occupation employment shares over time in the same format as Figure 1. Comparing these reveals that the trends in actual hours and in implied employment shares are very similar.

Finally, we calculate sectoral income shares as

\[ \Psi_J = \frac{\text{earnings of workers in sector } J}{\text{total earnings}}. \]

We use data from the U.S. Bureau of Economic Analysis (BEA) between 1960 and 2017 to get sectoral prices and the growth rate of GDP per full-time equivalent worker between periods. Table 4 in the Appendix contains all the calibration targets, and these are also plotted along with the model outcomes in section 4.

### 3.2 Extracting sector-occupation cell productivities

As mentioned before, given the structure of the model we can infer the productivity parameters using key equations from the model’s production side directly from the data, without having to rely on a parameterization of the model’s household side. We can do this conditional on a value for the elasticity of substitution in production between different types of labor, following similar steps as in Bárány and Siegel (2019).

We calculate the nine cell-specific productivity parameters, the \( \alpha_s \), in each period. We back these out directly from nine targets: the labor income share of different occupations within each sector, the relative sectoral prices, and the overall growth of the economy. We also take for now as given occupational wage rates, occupational labor supplies, and the sectoral distribution of income. However, the calibration of

---

11The industry classification system changed from SIC to NAICS in the middle of our sample, both systems are different from the classification used in the IPUMS Census/ACS. Table 3 in the Appendix shows the mapping of fine industries of each system into our broad sector categories.
the household side of our model guarantees that these are matched in general equilibrium in the initial and final period (1960 and 2017). Our model allows us to express the cell-specific productivity parameters as a function of the above data targets and the elasticity of substitution in production.

In particular, given occupational wages, the labor income share of different occupations within a sector pin down the ratios of $\alpha$ within sectors in each period from the firm’s optimality conditions (2) and (3):

$$\frac{\alpha_{mJ}}{\alpha_{rJ}} = \left(\frac{\theta_{mJ}}{\theta_{rJ}}\right)^{\frac{1}{\eta-1}} \frac{w_m}{w_r}, \quad (15)$$

$$\frac{\alpha_{aJ}}{\alpha_{rJ}} = \left(\frac{\theta_{aJ}}{\theta_{rJ}}\right)^{\frac{1}{\eta-1}} \frac{w_a}{w_r}. \quad (16)$$

The sectoral relative prices pin down (from (4)) the $\alpha$ across sectors within each period, again given occupational wages:

$$\frac{\alpha_{mJ}}{\alpha_{mK}} = \frac{p_K}{p_J} \left(\frac{\theta_{mJ}}{\theta_{mK}}\right)^{\frac{1}{\eta-1}}, \quad (17)$$

where we also used the expressions for the relative $\alpha$s within sectors.

Finally, the overall growth rate of output per (full-time-equivalent) worker pins down the evolution of the $\alpha$s over time, given the distribution of income across sectors and occupational labor supplies. Appendix A.2 shows the full derivations.

Thus, we have shown how to extract sector-occupation specific productivities from the data conditional on the elasticity of substitution across occupations. While there is no consensus on the value of this in the literature, there seems to be wide agreement that occupations are complementary, implying a value of $\eta$ less than one. To our knowledge the only estimate of this elasticity is in [Goos et al. (2014)], who estimate this for 21 occupations to be 0.53, 0.66, and 0.8 depending on the specification and the sample of countries; it is worth to note, however, that they estimate in partial equilibrium not taking into account aggregate effects. [Duennecker and Herrendorf (2016)] calibrate a value of 0.56 for 2 occupations, while [Lee and Shin (2017)] calibrate a value of 0.70, and [Aum et al. (2018)] a value of 0.81 for this same parameter for 10 occupa-
tions. With fewer, coarser occupations this elasticity is likely to be smaller, and hence we set $\eta = 0.6$ for our 3 occupations as our baseline, but conduct robustness checks around this value.

### 3.3 Calibration of the cost distribution and of the consumption side

To close the model we need to parameterize the household side. In calibrating the distribution of cost differences, we assume that $f(\chi_1, \chi_2)$ is a time-invariant bivariate normal distribution\(^\text{12}\) and we fix the correlation parameter, $\rho$, to be 0.4. Given this correlation, we calibrate the two means ($\mu_1, \mu_2$) and the diagonal elements of the variance-covariance matrix ($\sigma_1, \sigma_2$) such that in the initial and final period for given unit wages the cost distribution is able to match the employment shares.

Finally we calibrate the preference parameters of the model. Following Ngai and Pissarides (2007), we set the elasticity of substitution in consumption between the different sectoral outputs to $\varepsilon = 0.2$, implying that goods and the two types of services are complements. Given all the production side parameters and the distribution of costs we calibrate $c_L, c_H, a_L$, and $a_H$ (with $a_G = 1 - a_L - a_H$) to match the sectoral income shares in the initial and final year, i.e. in 1960 and 2017. This also guarantees that the relative occupational wages in 1960 and 2017 are met in equilibrium. Table 1 contains the calibrated parameters of the model which, together with the evolution of the $\alpha$s as backed out from the data, fully specify the calibrated model.

It is important to note that the occupational wages and the sectoral income shares are only matched by the model in the initial and final period; in between they are not matched, as these interim periods are not targeted in the calibration. However, the model does reasonably well in matching most statistics in all periods, see Figures 4 to 6.

We conduct robustness checks on the importance of the correlation parameter, $\rho$, and the elasticity of substitution in consumption, $\varepsilon$, and find neither to matter much; see Section 4.3. This is partly due to the calibration procedure, as the initial and final

\(^{12}\)For simplicity we assume that the distribution is time invariant. Allowing for changing costs (for example as in Caselli and Coleman (2001)) would require more parameters to be calibrated, and it would not affect the sector-occupation cell productivities, neither their decomposition into various components, nor the results from the baseline model.

15
<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$ elasticity of substitution in consumption</td>
<td>0.2</td>
</tr>
<tr>
<td>$\rho$ correlation of cost differences</td>
<td>0.4</td>
</tr>
<tr>
<td>$\mu_1, \mu_2$ mean of cost distribution</td>
<td>(0.0963, 0.4656)</td>
</tr>
<tr>
<td>$\sigma_1^2, \sigma_2^2$ variance of cost distribution</td>
<td>(0.1473, 0.0245)</td>
</tr>
<tr>
<td>$\eta$ elasticity of substitution in production</td>
<td>0.6</td>
</tr>
<tr>
<td>$\bar{c}_L$ non-homotheticity term in $L$</td>
<td>0.0034</td>
</tr>
<tr>
<td>$\bar{c}_H$ non-homotheticity term in $H$</td>
<td>0.0120</td>
</tr>
<tr>
<td>$a_L$ weight on $L$</td>
<td>0.00550</td>
</tr>
<tr>
<td>$a_H$ weight on $H$</td>
<td>0.99448</td>
</tr>
</tbody>
</table>

Period outcomes are guaranteed to be the same in the baseline across all calibrations.

4 The role of technological biases

In this section we first decompose the extracted cell level productivity growth paths into sector- and occupation-components, similarly to Bárány and Siegel (2019). We then use our general equilibrium model to quantify the role of each component of cell productivity growth for various outcomes of interest. It is important to note that in order to correctly assess the impact of various technological biases, we need to consider the endogenous response of sectoral demands and of occupational labor supplies to technological change.

4.1 Decomposition of technological change

We decompose the extracted series of sector-occupation specific productivities into sector-specific and occupation-specific components using a factor model. In particular, we run the following regression on the log difference of the cell productivities,

$$\Delta \ln \alpha_{o,J,t} \equiv \ln \alpha_{o,J,t} - \ln \alpha_{o,J,t-1} = \beta_t + \gamma_{J,t} + \delta_{o,t} + \varepsilon_{o,J,t}.$$  \hfill (18)
In the regression we use each cell’s average labor income share between period \( t - 1 \) and \( t \) as weights \( \omega_{oJ,t} \) to reflect the relative importance of the sector-occupation cell. We restrict the average sector effect \( \left( \sum_o \sum_J \omega_{oJ,t} \gamma_{J,t} \right) \) and the average occupation effect \( \left( \sum_o \sum_J \omega_{oJ,t} \delta_{o,t} \right) \) across all cells to be zero. These restrictions imply that \( \beta_t \), the time effect, is the average technological change between period \( t - 1 \) and \( t \) in all cells. The time-varying sector effects, \( \gamma_{J,t} \), capture sector-wide innovations that affect the labor productivity of all workers in that sector equally regardless of their occupation. Productivity changes that are common to workers of a given occupation, but are independent from the sector, are assigned to the time-varying occupation effects, \( \delta_{o,t} \). The residual \( \varepsilon_{oJ,t} \) reflects productivity changes idiosyncratic to workers in a sector-occupation cell. The \( R^2 \) of this regression is 89.4\% meaning that neutral-, sector- and occupation-specific components jointly describe the evolution of cell productivities very well, and that only around 10\% of the variation is idiosyncratic to the sector-occupation cell.

To assess how much of the evolution of cell productivities is explained by the sector-specific and the occupation-specific components respectively, we build counterfactual cell productivity series which in turn shut down various components. All series are constructed starting from the extracted initial cell productivity \( \ln \alpha_{Jo,t} \). We then add to this in turn counterfactual series for \( \Delta \ln \alpha_{Jo,t} \) constructed from neutral and sector-specific components only (‘sector-only’ \( \hat{\beta}_t + \hat{\gamma}_{J,t} \)), from neutral and occupation-specific components only (‘occupation-only’ \( \hat{\beta}_t + \hat{\delta}_{o,t} \)), and from neutral, sector- and occupation-specific components (‘full factor’ \( \hat{\beta}_t + \hat{\gamma}_{J,t} + \hat{\delta}_{o,t} \)).

When evaluating the explanatory power of the sector-only and the occupation-only predictions, it is important to bear in mind that these series are not equivalent to the predictions of a factor model with a time and a sector or respectively a time and an occupation component only. Those productivity series would pick up differential productivity growth across sectors (or occupations) that originates from the sectors using occupations at different intensities (or the occupations being used at different intensities across sectors).

\[13\]To be precise, we use as weights \( \omega_{oJ,t} = (\Psi_{J,t} \theta_{oJ,t} + \Psi_{J,t-1} \theta_{oJ,t-1})/2 \), where the values are given in Appendix Table[4]. The results are very robust to alternatives, such as using employment shares, or using year \( t - 1 \) or year \( t \) shares, rather than averages.
In Figure 3 we show the path of cell productivities as extracted from the data, as well as the different predicted productivities as explained above. The red solid lines show the baseline cell productivities, the green solid lines the full factor prediction, the blue dashed lines the sector-only, while the yellow dashed-dotted lines occupation-only counterfactuals. The figure shows that the full factor and the occupation-only predictions are quite close to each other and to the baseline, whereas the sector-only predictions are further away.

### 4.2 The role of technologies in equilibrium outcomes

We now study the role of the sector- and occupation-specific components in the evolution of various outcomes in our general equilibrium model. Figure 4 shows occupational employment and wages, Figure 5 sectoral employment and prices, and Figure 6 occupational income shares within sectors. In all figures we show the evolution of the data in solid grey, contrasted with the model’s predictions for the various counterfactual productivity paths color coded as before: the baseline in solid red, the full factor in solid green, the sector-only in dashed blue, and occupation-only in dashed-dotted yellow lines.

The first thing to note in Figures 4 to 6 is that our baseline model does very well in matching the data. It is important to recall that our baseline model matches the
data exactly in the initial and final period by construction, but not in the interim periods. Nonetheless, even in the interim periods the differences with the data turn out to be small for almost all outcomes of interest, implying that in all periods the baseline model’s predictions are extremely close to the data except for occupational relative wages, where they do not pick up the (short-lived) drop in the 1980 values.\footnote{Our model’s failure to match these paths can be understood by looking at Figure 4b, where the dark gray solid line shows the data and the red solid line the values predicted by our baseline model. The data, as our model, displays a strong upward trend in both manual and abstract wages relative to routine. In the data, however the 1980 values of these relative wages seem to be outliers, which might correspond to the compression of the skill premium during the 1970s. Our model stays silent about what generated these.}

![Graphs showing occupational outcomes](image)

**Figure 4**: The evolution of occupational outcomes

The second thing to note is that the model based on the productivity growth predictions of the full factor model does almost as well as our baseline model. The only difference between these two models is that the latter does not contain the productivity growth that is idiosyncratic to the sector-occupation cell. The fact that these two models perform equally well for occupational employment and wages, for sectoral employment and for sectoral prices implies that the productivity growth component idiosyncratic to the cell is not the key driver of these outcomes. However, for occupational income shares within sectors ($\theta_{o,j}$), there is some discrepancy between the data...
(and thus the baseline model) and the full factor model prediction, highlighting that productivity growth idiosyncratic to the sector-occupation cell plays a more important role in this outcome.

Before analyzing in detail the predictions based on the sector components and the occupation components alone, it is worth to point out what channels operate in our general equilibrium model. The first channel is that relative technologies within a sector impact optimal relative occupational labor demand within the sector, as shown in (2) and (3). The second channel is that all technologies within a sector impact the sectoral prices, as in (4). Changes in prices affect the sectoral consumption demands through (11), (12) and (13). In turn, this impacts how much employment needs to reallocate across sectors. Together with the first channel this determines by how occupational labor demands and thus market-clearing occupational wages change. Technological change due to occupation components works through both of these channels, whereas sector-specific technological change, which scales all \( \alpha_{o,J} \) within sector \( J \) by the same factor, does not alter the relative labor demands within a sector.

![Graphs of L, G, and H over time](image1)

![Graphs of L/G and H/G over time](image2)

Figure 5: The evolution of sectoral outcomes

Figure 4 shows (i) that the predictions of the full factor and the occupation-only
models are very close to each other and to the data for all occupational outcomes, and (ii) that while the sector-only model’s predictions are qualitatively in line with the data, quantitatively they fall short. Figure 5 shows (i) that the full factor model does almost as well as our baseline model, (ii) that for sectoral employment both the occupation-only and the sector-only models are qualitatively and quantitatively close to the data, and (iii) that for sectoral prices neither the occupation-only nor the sector-only model does very well.\footnote{Qualitatively the path for sectoral income shares in the data and in the various counterfactuals are very similar to those of sectoral employment shares.}

Figure 4 and 5a demonstrate that both the occupation components and the sector components by themselves generate occupational employment and wage as well as sectoral employment paths qualitatively in line with the data. However, it is evident that the occupation-component plays a much larger role for labor market outcomes.

That the occupation-specific component implies effects very similar to the full factor prediction is not surprising, given that Figure 3 already established that the occupation factors mimic most of the evolution of sector-occupation cell productivities. The sector-specific component in technological change generates dynamics consistent with the data as it induces shifts in consumption demands towards the service sectors, and thus in sectoral employment, i.e. structural transformation in line with the data. Since the high-skilled service sector is the most intensive in abstract and the low-skilled service sector in manual occupations, these sectoral shifts also lead to a decline in the relative demand for routine occupations, leading to the polarization of occupational employment and wages. However, as relative productivities across occupations within a sector have not changed, sector-specific technological change does not induce a decline of routine employment within a sector.\footnote{In fact, as \( \frac{\bar{v}}{w_r} \) and \( \frac{\bar{v}}{w_a} \) increase, it leads to a small (and counterfactual) rise in the routine employment share of each sector.} The sector-component therefore understates the overall changes in occupational outcomes. Moreover, since some of the observed changes in relative prices are due to occupation-specific technological change (as established by \footnote{Qualitatively the path for sectoral income shares in the data and in the various counterfactuals are very similar to those of sectoral employment shares.}), it also somewhat understates the sectoral reallocations.

Figure 6 shows the predicted changes in labor income shares within sectors. This figure shows that the sector-only component predicts hardly any change in the \( \theta_s \).
This can be understood from equations (19) and (20) in the Appendix: labor income shares change if relative occupational wages or relative cell productivities within a sector change. The sector-only model shuts down the second channel, and predicts quantitatively small changes in relative occupational wages, thus implying changes in the $\theta$s that are in line with the data, but which are quantitatively very small. The figure also reveals that in general the implications of the occupation-only and of the full-factor productivity series are very close to each other and the data. However, for manual occupations and low-skilled services, there are larger discrepancies which highlight the role of the idiosyncratic cell productivity components in the evolution of $\theta_{oJ}$.

![Graph](Figure 6: Income shares within sectors)

4.3 Robustness checks

In this section we explore the robustness of our results to alternative values of the elasticity of substitution in production, $\eta$, in consumption, $\varepsilon$, and to alternative correlation parameters in the occupational entry costs, $\rho$. In each of these robustness checks we recalibrate the model based on the alternative values of the fixed parameters we consider. Note that from these three parameters, only the value of $\eta$ matters for the

---

17The behavior of the sector-occupation cell employment shares in response to the evolution of the various productivity components is very similar to Figure 6 and not shown for brevity.
sector-occupation technologies that we infer from the data as these are fully pinned down by the production side of the model. However, all three parameters impact the general equilibrium outcomes of the model.

Table 2 summarizes the impact of changing each of these three parameters, one by one, on our results. We report for each outcome the average of the fraction of the observed change between 1960 and 2017 explained by the model for various counterfactual productivity series:

$$\left(\frac{\Delta x^m}{\Delta x^d}\right),$$

where $\Delta x^j$ is the change between 1960 and 2017 in our outcome of interest, in the data for $j = d$ and in the model for $j = m$. We average over $\{m, r, a\}$ for occupational employment shares, over $\{m, a\}$ for occupational wages relative to $r$, over $\{L, G, H\}$ for sectoral employment shares, over $\{L, H\}$ for sectoral prices relative to $G$, and over $\{m, r, a\} \times \{L, G, H\}$ for occupational income shares within sectors. A value of 100 means the model perfectly replicates the average change in the data, whereas smaller numbers indicate that on average the model prediction generates the changes only partially and numbers above 100 that the model overstates the actual change. The top panel shows our baseline calibration, with each row representing a different productivity path (extracted (base), full factor model (full), sector only (sec) and occupation only (occ)). Each column shows the fraction explained of an outcome of interest, as shown in Figures 4–6. The subsequent panels show the same for alternative parameterizations. By construction in each panel the top row (using the extracted productivity paths) explains 100 percent of the 1960 to 2017 changes. This table shows that all our predictions are maintained qualitatively, and are quite robust quantitatively. The full factor and the occupation-only predictions in each panel are quite close to 100 and to each other, except for sectoral prices, where both over-predict changes substantially. Our finding that predictions based on the sector components alone qualitatively are in line with the data, but quantitatively are quite far, bares out in the robustness checks. Also, the full factor and the occupation-only predictions explain 84 percent of the change in cell income shares across all specifications, showing that productivity growth idiosyncratic to sector-occupation cells plays a role in these outcomes. The
only outcome which is quantitatively sensitive to $\eta$ are relative sectoral prices. The larger the value for $\eta$ is, the further all (including the sector-only) counterfactual predictions get from the actual change in sectoral relative prices. This implies that the role of sector-occupation specific productivity growth is more pronounced for higher elasticities of substitutions. The only outcome sensitive to the elasticity of substitution in consumption, $\varepsilon$, is sectoral employment. This is most pronounced for the sector-only predictions: for smaller elasticities the sector-only component over-predicts the changes in sectoral employment, and for larger elasticities, it under-predicts it. None of our results are sensitive to the correlation of occupational entry costs.
### Table 2: Explained 1960–2017 changes under alternative parameters

<table>
<thead>
<tr>
<th></th>
<th>Outcomes</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>occupational</td>
<td>sectoral</td>
<td>cell</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>empl. rel. wages</td>
<td>empl. rel. prices</td>
<td>(\theta)</td>
<td></td>
</tr>
<tr>
<td><strong>Baseline:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\eta = 0.6, \rho = 0.4, \varepsilon = 0.2)</td>
<td>base</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>104</td>
<td>102</td>
<td>105</td>
<td>164</td>
</tr>
<tr>
<td></td>
<td>sec</td>
<td>36</td>
<td>22</td>
<td>95</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>occ</td>
<td>106</td>
<td>104</td>
<td>97</td>
<td>166</td>
</tr>
<tr>
<td><strong>Alternative (\eta):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\eta = 0.5)</td>
<td>base</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>103</td>
<td>102</td>
<td>103</td>
<td>151</td>
</tr>
<tr>
<td></td>
<td>sec</td>
<td>41</td>
<td>25</td>
<td>95</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>occ</td>
<td>103</td>
<td>102</td>
<td>98</td>
<td>125</td>
</tr>
<tr>
<td>(\eta = 0.7)</td>
<td>base</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>104</td>
<td>102</td>
<td>109</td>
<td>187</td>
</tr>
<tr>
<td></td>
<td>sec</td>
<td>29</td>
<td>17</td>
<td>91</td>
<td>-8</td>
</tr>
<tr>
<td></td>
<td>occ</td>
<td>110</td>
<td>108</td>
<td>95</td>
<td>246</td>
</tr>
<tr>
<td><strong>Alternative (\varepsilon):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\varepsilon = 0.1)</td>
<td>base</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>104</td>
<td>101</td>
<td>110</td>
<td>164</td>
</tr>
<tr>
<td></td>
<td>sec</td>
<td>31</td>
<td>16</td>
<td>127</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>occ</td>
<td>107</td>
<td>106</td>
<td>91</td>
<td>166</td>
</tr>
<tr>
<td>(\varepsilon = 0.3)</td>
<td>base</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>103</td>
<td>102</td>
<td>99</td>
<td>164</td>
</tr>
<tr>
<td></td>
<td>sec</td>
<td>43</td>
<td>29</td>
<td>48</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>occ</td>
<td>105</td>
<td>103</td>
<td>105</td>
<td>165</td>
</tr>
<tr>
<td><strong>Alternative (\rho):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\rho = 0)</td>
<td>base</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>104</td>
<td>102</td>
<td>105</td>
<td>164</td>
</tr>
<tr>
<td></td>
<td>sec</td>
<td>37</td>
<td>21</td>
<td>95</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>occ</td>
<td>106</td>
<td>104</td>
<td>97</td>
<td>166</td>
</tr>
<tr>
<td>(\rho = 0.8)</td>
<td>base</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>103</td>
<td>102</td>
<td>105</td>
<td>164</td>
</tr>
<tr>
<td></td>
<td>sec</td>
<td>34</td>
<td>26</td>
<td>94</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>occ</td>
<td>106</td>
<td>105</td>
<td>97</td>
<td>166</td>
</tr>
</tbody>
</table>
5 Conclusion

In this paper we use a general equilibrium model to infer and study the consequences of biased technological change in labor market reallocations. Drawing on key firm side equations we infer sector-occupation-specific technologies from the data, which we then decompose into sector- and occupation-specific components, as well as sector-occupation cell specific residuals. This decomposition shows that sector- and occupation-specific components jointly explain about 90 percent of the variation in cell level productivity growth. We evaluate the role of these components by feeding these as counterfactual productivity series into the model. We find that both the sector-bias and the occupation-bias in technological change are causes of structural transformation, the observed sectoral reallocation of employment, as well as of polarization of occupational employment and wages. However, quantitatively our findings indicate a major role for occupation-specific components. Moreover we find that the occupation components and the cell-specific elements are important drivers of the occupational income shares within sectors. To explain the evolution of sectoral prices over time both sector and occupation components are needed.

While our model does not allow for any frictions and therefore does not warrant any policy interventions, the finding that virtually all of labor market outcomes are explained by the occupation component suggests that if policymakers wanted to respond to the observed reallocations, they should not focus on industrial policies but consider active labor market policies, including training programs.

References


A Appendix

A.1 Classification

The classification of workers into occupational categories and into industrial sectors is identical to the assignments in Bárány and Siegel (2019). For ease of reference, these mappings are listed below.

We combine three different industry classification systems, the NAICS, the SIC, and the ind1990. Table 3 summarizes our categorization in terms of each system.

<table>
<thead>
<tr>
<th>NAICS</th>
<th>SIC</th>
<th>ind1990</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-skilled services</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wholesale trade</td>
<td>Wholesale trade</td>
<td>Wholesale &amp; retail trade</td>
</tr>
<tr>
<td>Retail trade</td>
<td>Retail trade</td>
<td></td>
</tr>
<tr>
<td>Transportation &amp; warehousing</td>
<td>Transportation</td>
<td>Transportation</td>
</tr>
<tr>
<td>Arts, entertainment, recreation, accommodation</td>
<td>Amusement &amp; recreation serv.</td>
<td>Entertainment</td>
</tr>
<tr>
<td>&amp; food serv.</td>
<td>Motion pictures</td>
<td></td>
</tr>
<tr>
<td>Other serv., except government</td>
<td>Hotels &amp; other lodging places</td>
<td>Low-skilled business serv.</td>
</tr>
<tr>
<td></td>
<td>Personal serv.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Auto repair, serv. &amp; parking</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Miscellaneous repair serv.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Private households</td>
<td>Personal serv.</td>
</tr>
<tr>
<td></td>
<td>Other serv., except government</td>
<td></td>
</tr>
<tr>
<td>Goods</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agriculture, forestry, fishing &amp; hunting</td>
<td>Agriculture, forestry, &amp; fishing</td>
<td>Agriculture, forestry &amp; fishing</td>
</tr>
<tr>
<td>Mining</td>
<td>Mining</td>
<td>Mining</td>
</tr>
<tr>
<td>Construction</td>
<td>Construction</td>
<td>Construction</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Manufacturing</td>
<td>Manufacturing</td>
</tr>
<tr>
<td>High-skilled services</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Utilities</td>
<td>Electric, gas, &amp; sanitary serv.</td>
<td>Utilities</td>
</tr>
<tr>
<td>Information</td>
<td>Communications</td>
<td>Communications</td>
</tr>
<tr>
<td>Finance, insurance, real estate, rental &amp;</td>
<td>Finance, insurance, &amp; real estate</td>
<td>Finance, insurance &amp; real estate</td>
</tr>
<tr>
<td>leasing</td>
<td>Legal serv.</td>
<td></td>
</tr>
<tr>
<td>Educational serv., health care &amp; social</td>
<td>Miscellaneous professional serv.</td>
<td>High-skilled business serv.</td>
</tr>
<tr>
<td>assistance</td>
<td>Membership organizations</td>
<td></td>
</tr>
<tr>
<td>Government</td>
<td>Educational serv.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Health serv.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Social serv.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Government</td>
<td>Public administration</td>
</tr>
</tbody>
</table>

We classify occupations based on their routine task content and cognitive requirements, similarly to Acemoglu and Autor (2011), into the following three categories:

**Manual** (low-skilled non-routine): housekeeping, cleaning, protective service, food preparation and service, building, grounds cleaning, maintenance, personal appear-
ance, recreation and hospitality, child care workers, personal care, service, healthcare support;

**Routine**: farm workers, construction trades, extractive, machine operators, assemblers, inspectors, mechanics and repairers, precision production, transportation and material moving occupations, sales, administrative support;

**Abstract** (skilled non-routine): managers, management related, professional specialty, technicians and related support.

### A.2 Derivations

In this subsection we show how the $\alpha$s can be expressed as a function of observables.

The labor income shares of different occupations within a sector pin down the $\alpha$s within a sector. To see this multiply (2) with $w_m/w_r$ and (3) with $w_a/w_r$ to get:

\[
\frac{\theta_{mJ}}{\theta_{rJ}} = \left( \frac{w_r}{w_m} \right)^{\eta-1} \left( \frac{\alpha_{mJ}}{\alpha_{rJ}} \right)^{\eta-1},
\]

(19)

\[
\frac{\theta_{aJ}}{\theta_{rJ}} = \left( \frac{w_r}{w_a} \right)^{\eta-1} \left( \frac{\alpha_{aJ}}{\alpha_{rJ}} \right)^{\eta-1}.
\]

(20)

Re-arrange to get:

\[
\frac{\alpha_{mJ}}{\alpha_{rJ}} = \left( \frac{\theta_{mJ}}{\theta_{rJ}} \right)^{\frac{1}{\eta-1} w_m},
\]

\[
\frac{\alpha_{aJ}}{\alpha_{rJ}} = \left( \frac{\theta_{aJ}}{\theta_{rJ}} \right)^{\frac{1}{\eta-1} w_a}.
\]

The relative prices across sectors pin down the relative $\alpha$s across sectors. To see this, use (4) for sectors $J$ and $K$ to get:

\[
\frac{p_J}{p_K} = \frac{\alpha_{mK}}{\alpha_{mJ}} \left[ \frac{1}{w_m^0} + \left( \frac{\alpha_{rK}}{\alpha_{mK}} \right)^{\eta-1} \frac{1}{w_r^0} + \left( \frac{\alpha_{aK}}{\alpha_{mK}} \right)^{\eta-1} \frac{1}{w_a^0} \right]^{-\frac{1}{\eta-1}},
\]

30
Using the above expressions on the relative \( \alpha \)s within sector and re-arranging we get:

\[
\frac{\alpha_{mJ}}{\alpha_{mK}} = \frac{p_K}{p_J} \left( \frac{\theta_{mK}}{\theta_{mJ}} \right)^{\frac{1}{1-\eta}}.
\]

The growth rate of the economy pins down the evolution of the \( \alpha \)s over time. First, note that we express the evolution of cell productivities over time conditional on the sectoral income shares. The sectoral income shares, using equations (5), (6) and (7), can be expressed as:

\[
\frac{\Psi_G}{\Psi_H} = \frac{p_G Y_G}{p_H Y_H} = \frac{l_m G \theta_{mG}^{1-\eta} w_m^{\eta} \alpha_{mG}^{1-\eta}}{l_m H \theta_{mH}^{1-\eta} w_m^{\eta} \alpha_{mH}^{1-\eta}},
\]

\[
\frac{\Psi_L}{\Psi_H} = \frac{p_L Y_L}{p_H Y_H} = \frac{l_m L \theta_{mL}^{1-\eta} w_m^{\eta} \alpha_{mL}^{1-\eta}}{l_m H \theta_{mH}^{1-\eta} w_m^{\eta} \alpha_{mH}^{1-\eta}}.
\]

Re-arranging and using the above expressions to substitute out \( \alpha_{mJ}/\alpha_{mK} \):

\[
\frac{l_m G}{l_m H} = \frac{\Psi_G}{\Psi_H} \left( \frac{p_H}{p_G} \right)^{1-\eta} \left( \frac{\alpha_{mH}}{\alpha_{mG}} \right)^{1-\eta} = \frac{\Psi_G \theta_{mG}}{\Psi_H \theta_{mH}},
\]

\[
\frac{l_m L}{l_m H} = \frac{\Psi_L}{\Psi_H} \left( \frac{p_H}{p_L} \right)^{1-\eta} \left( \frac{\alpha_{mH}}{\alpha_{mL}} \right)^{1-\eta} = \frac{\Psi_L \theta_{mL}}{\Psi_H \theta_{mH}}.
\]

Using that \( l_m L + l_m G + l_m H = l_m \), we can express

\[
l_m H = \frac{l_m}{\frac{\Psi_L \theta_{mL}}{\Psi_H \theta_{mH}} + \frac{\Psi_G \theta_{mG}}{\Psi_H \theta_{mH}} + 1}.
\]

We can express sector-\( H \) price as a function of observables by plugging (19) and (20) into (4), and using that the \( \theta \)s sum to 1 within sector:

\[
p_H = \left[ \left( \frac{\alpha_{mH}}{w_m} \right)^{\eta-1} \left( \frac{\alpha_{rH}}{w_r} \right)^{\eta-1} \left( \frac{\alpha_{aH}}{w_a} \right)^{\eta-1} \right]^{\frac{1}{1-\eta}} = \frac{w_m}{\alpha_{mH}} \left( \frac{1}{\theta_{mH}} \right)^{\frac{1}{1-\eta}}.
\]

Similarly using (2), (3) and relative \( \alpha \)s within sectors as expressed above, as well as
that within sectors the $\theta$s sum to 1, sectoral output can be expressed as:

$$
Y_L = \left[ (\alpha_{mL} l_mL)^{\frac{-1}{\eta}} + (\alpha_{rL} r_rL)^{\frac{-1}{\eta}} + (\alpha_{aL} a_aL)^{\frac{-1}{\eta}} \right]^{\frac{1}{\eta-1}} 
= \alpha_{mL} l_mL \left[ 1 + \left( \frac{\alpha_{rL} r_rL}{\alpha_{mL} l_mL} \right)^{\frac{-1}{\eta}} + \left( \frac{\alpha_{aL} a_aL}{\alpha_{mL} l_mL} \right)^{\frac{-1}{\eta}} \right]^{\frac{1}{\eta-1}} = \alpha_{mL} l_mL \left( \frac{1}{\theta_{mL}} \right)^{\frac{1}{\eta-1}}
$$

$$
Y_G = \frac{\alpha_{mH} l_mH}{\theta_{mL}} \frac{p_H}{p_L} \Psi_G
$$

$$
Y_H = \frac{\alpha_{mH} l_mH}{\theta_{mL}} \theta_{mH} \frac{p_H}{p_G} \Psi_H.
$$

Using the above and the expressions for $p_H$ and $l_{mH}$ we can then write the value of output at current prices as:

$$
p_L Y_L + p_G Y_G + p_H Y_H = w_m l_m \frac{\Psi_L}{\Psi_H} \frac{\Psi_G}{\Psi_H} + 1
$$

We can express the value of output at initial prices, where we denote by 0 the initial period and we omit the subscript $t$ in all other periods for brevity, as:

$$
p_{L,0} Y_L + p_{G,0} Y_G + p_{H,0} Y_H
= \frac{\alpha_{mH}}{\alpha_{mH,0}} w_{m,0} \left( \frac{\theta_{mH,0}}{\theta_{mH,0}} \right)^{\frac{1}{\eta-1}} l_{m,0} \frac{\Psi_L}{\Psi_H} \frac{\Psi_G}{\Psi_H} + \theta_{mH,0} \frac{p_{L,0} \Psi_L}{p_{H,0} \Psi_H} + \frac{p_{G,0} \Psi_G}{p_{H,0} \Psi_H} + 1
$$

The equivalent of output growth in our model is:

$$
1 + \gamma = \frac{p_{L,0} Y_L + p_{G,0} Y_G + p_{H,0} Y_H}{p_{L,0} Y_{L,0} + p_{G,0} Y_{G,0} + p_{H,0} Y_{H,0}}.
$$

The evolution of $\alpha_{mH}$ over time is therefore pinned down by:

$$
\frac{\alpha_{mH}}{\alpha_{mH,0}} = \left( \frac{\theta_{mH}}{\theta_{mH,0}} \right)^{\frac{1}{\eta-1}} l_{m,0} \frac{\Psi_L}{\Psi_H} \frac{\Psi_G}{\Psi_H} + \theta_{mH,0} \frac{p_{L,0} \Psi_L}{p_{H,0} \Psi_H} + \frac{p_{G,0} \Psi_G}{p_{H,0} \Psi_H} + 1
$$
A.3 Calibration targets

Table 4 contains the targets used in the calibration.

Table 4: Calibration targets

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_L/p_G$</td>
<td>0.9360</td>
<td>1.0651</td>
<td>0.8604</td>
<td>0.9177</td>
<td>0.9658</td>
<td>0.9834</td>
<td>1.0369</td>
</tr>
<tr>
<td>$p_H/p_G$</td>
<td>0.4882</td>
<td>0.5769</td>
<td>0.5110</td>
<td>0.7301</td>
<td>0.9472</td>
<td>0.9831</td>
<td>1.0261</td>
</tr>
<tr>
<td>$\Psi_L$</td>
<td>0.2563</td>
<td>0.2606</td>
<td>0.2604</td>
<td>0.2747</td>
<td>0.2794</td>
<td>0.2805</td>
<td>0.2751</td>
</tr>
<tr>
<td>$\Psi_G$</td>
<td>0.4734</td>
<td>0.4121</td>
<td>0.3800</td>
<td>0.3068</td>
<td>0.2624</td>
<td>0.2120</td>
<td>0.2095</td>
</tr>
<tr>
<td>$\Psi_H$</td>
<td>0.2703</td>
<td>0.3274</td>
<td>0.3596</td>
<td>0.4186</td>
<td>0.4582</td>
<td>0.5075</td>
<td>0.5155</td>
</tr>
<tr>
<td>growth</td>
<td>1</td>
<td>1.2009</td>
<td>1.3410</td>
<td>1.5482</td>
<td>1.8230</td>
<td>2.1971</td>
<td>2.2489</td>
</tr>
<tr>
<td>$w_m/w_r$</td>
<td>0.6231</td>
<td>0.6993</td>
<td>0.7520</td>
<td>0.7947</td>
<td>0.8368</td>
<td>0.8335</td>
<td>0.8382</td>
</tr>
<tr>
<td>$w_a/w_r$</td>
<td>1.1936</td>
<td>1.2066</td>
<td>1.1438</td>
<td>1.2320</td>
<td>1.2969</td>
<td>1.4243</td>
<td>1.4370</td>
</tr>
<tr>
<td>$\theta_{mL}$</td>
<td>0.1134</td>
<td>0.104</td>
<td>0.1183</td>
<td>0.126</td>
<td>0.1465</td>
<td>0.1697</td>
<td>0.1698</td>
</tr>
<tr>
<td>$\theta_{rL}$</td>
<td>0.6827</td>
<td>0.6565</td>
<td>0.65</td>
<td>0.616</td>
<td>0.5522</td>
<td>0.4931</td>
<td>0.4748</td>
</tr>
<tr>
<td>$\theta_{aL}$</td>
<td>0.2039</td>
<td>0.2395</td>
<td>0.2317</td>
<td>0.258</td>
<td>0.3013</td>
<td>0.3371</td>
<td>0.3554</td>
</tr>
<tr>
<td>$\theta_{mG}$</td>
<td>0.0116</td>
<td>0.0177</td>
<td>0.0186</td>
<td>0.0202</td>
<td>0.0192</td>
<td>0.0236</td>
<td>0.0249</td>
</tr>
<tr>
<td>$\theta_{rG}$</td>
<td>0.7887</td>
<td>0.7501</td>
<td>0.7419</td>
<td>0.6535</td>
<td>0.6184</td>
<td>0.5554</td>
<td>0.5421</td>
</tr>
<tr>
<td>$\theta_{aG}$</td>
<td>0.1998</td>
<td>0.2323</td>
<td>0.2395</td>
<td>0.3264</td>
<td>0.3624</td>
<td>0.4211</td>
<td>0.4330</td>
</tr>
<tr>
<td>$\theta_{mH}$</td>
<td>0.1051</td>
<td>0.1064</td>
<td>0.1082</td>
<td>0.0915</td>
<td>0.0863</td>
<td>0.0866</td>
<td>0.0788</td>
</tr>
<tr>
<td>$\theta_{rH}$</td>
<td>0.4197</td>
<td>0.3709</td>
<td>0.3526</td>
<td>0.3075</td>
<td>0.2508</td>
<td>0.2134</td>
<td>0.1927</td>
</tr>
<tr>
<td>$\theta_{aH}$</td>
<td>0.4752</td>
<td>0.5226</td>
<td>0.5392</td>
<td>0.6010</td>
<td>0.6629</td>
<td>0.7000</td>
<td>0.7284</td>
</tr>
</tbody>
</table>

Figure 7 is the model analogue to Figure 1 where employment shares are inferred from income shares and wages per efficiency unit, as described in Section 3.

Figure 7: Implied employment shares 1960-2017